# Stochastic optimal control in finance

# under impulsive changes and regime switches

## *- foundations, fresh ideas, discussions and friendship -*

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In this presentation, we give a short introduction into how we as academicians and practitioners might approach decision making in time as far as it is related to the financial sector. In fact, we may have in mind the decision making of individuals, of companies and, eventually, of economies.

Whenever we begin such a project, then there needs to be a so-called "modeling" of financial dynamics first, which has to be based on experience and data. After this first step, the decision making starts. In both steps, we fruitfully employ Optimization theory. This optimization might appear, at a first glance, as abstract and of a high level, but it essentially follows what we all study and learn at our universities, just in a way so that the time-dependence and the "uncertainty" of our financial processes are well included into our optimization of what we call "utility". We name this optimization along of time as "Optimal Control".

Herewith, we will study that financial dynamics which we called Stochastic Differential Equations, and then the optimization which we call Stochastic Optimal Control, subject to these dynamic constraints.

The right-hand side of a stochastic differential equation is the sum of several characteristic terms: (i) a Drift term (which is the "deterministic" part), (ii) a Diffusion term (which comprises a core part of the "random fluctuation" and has the famous Brownian motions in its center), and also (iii) an Impulsive term (which stands for the stochastic "jumps" that we shall see in the solutions of the stochastic differential equation). The famous Lévy processes unify both Brownian motions and jump processes.

Such differential equations appear as a limiting case, when we start with the famous One-Period Model of Markowitz (many listers will know it), then turn to the Multi-Period Model with its rebalancing at the beginning of each period, and finally make the time intervals smaller and smaller, obtaining a Continuous-Time Model.

In this presentation, we shall go a step further by allowing that not only solution curves of stochastitic differential equations “jump”, but that the whole “system” of such an equation jumps. In finance and economics, we sometimes call this a “Regime Switch”; in the fields of culture and civilization, we call it a “Paradigm Shift”.

When we name such processes and their equations as “Hybrid Systems”, then we mean by this a “mixed” system, a mixture between (a) continuity and (b) discreteness (combinatorics, impulsiveness) of the world in which we live. Those systems were proposed by scientists from Electrical and Electronics Engineering, they were used by us already in Biology and Bioinformatics (please think of the enormous role of mutations for the genetic information). We, with great care, apply Hybrid Systems in the financial world and in economics.

These systemes are very important but also very challenging phenomena. With our presentation we aim to discuss and understand their core facts, and to offer answers on how we can make financial and economical decisions in a world full of randonmess and full of sudden changes in the “worldviews”, triggered by catastrophs and disasters, by unexpected changes in elections and by revolutions of all kinds, etc.

Very shortly we shall mention on extensions of our study in the presence of Delays (a first form of Memory) and of Insider Information.

This presentation will be a lively one: the audienced will be addressed, their knowledge, expectations and wishes explored by the presenter, and a huge technical apparatus will be avoided – just the main ideas worked out. By no means we shall try to speak about “everything”. In fact, we shall meet with mathematics as someting very meaningful and useful in today’s world, also as something enjoyable, beautiful and entertaining, and something to work with as a team: during the seminar and, why not, in the future as friends.